

Some Determinants of Corporate Risk Aversion

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In this paper we roughly quantify the degree of risk aversion induced by three rationales for corporate risk management: the cost of financial distress, costly external finance, and the principal-agent relationship between shareholders and management. In so doing, we provide a foundation for the use of corporate utility functions. However, we are unable to fully support the degree of risk aversion reported in the decision analysis literature. Specifically, financial distress and costly external finance appear to induce relatively little risk aversion, while principal-agent concerns lend only partial support to published corporate risk tolerance guidelines.

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1. Introduction

There is, of course, an important sense in which preferences being entirely subjective cannot be in error; but in a different more subtle sense they can be.

—L. J. Savage (1954, p. 103)

The disciplines of corporate finance and decision analysis differ in how they characterize corporate risk preference. Corporate finance specifies the preferences, time and risk, that a corporation should use, whereas decision analysts have tended to view corporate risk preference as subjective.

1.1. Corporate Finance

Corporate finance starts with the premise that the corporate objective is to maximize the wealth of its shareholders. Then, based on the capital asset pricing model (CAPM) and the distinction between systematic and unsystematic (or diversifiable) uncertainties, corporate finance concludes that corporations should (in a normative sense) use a market-determined rate to discount systematic uncertainties and value diversifiable uncertainties at their expected value, discounted at the risk-free rate.¹ This implies that diversification by the firm is at best useless and at worst wasteful (Brealey and Myers 1991). Still, executives

rank risk management as a top priority (Rawls and Smithson 1990) and make hedging decisions consistent with their company's risk preferences (Lewent and Kearney 1990), even though these risk preferences are never defined or, at least, not defined in terms of a corporate utility function.

Finance theorists and practitioners have identified several rationales for risk management. Among these are convexity of the corporate tax code, direct and indirect costs of financial distress, costly external finance, and principal-agent problems between shareholders and management.

A progressive corporate income tax (increasing marginal tax) results in the expected income tax exceeding the tax on expected income (Mayers and Smith 1982, Smith and Stulz 1985). Graham and Smith (1999) demonstrate that this effect is minor. For example, of those firms that face a convex tax function, the average savings from a 5% reduction in income volatility is less than \$125,000 per year. Furthermore, three-quarters of firms would accrue no benefit by using hedging to reduce taxable income. Therefore, we will not consider this rationale further.

Financial distress creates adverse incentives between bondholders and shareholders (Jensen and Meckling 1976, Myers 1977) and between shareholders and customers (Titman 1984). This misalignment in incentives imposes real costs upon the firm, irrespective of

¹ See Brealey and Myers (1991) or Luenberger (1998) for a discussion of the CAPM and its implications.

whether distress was induced by systematic or unsystematic uncertainties. The objective of the firm can be modeled as maximizing its expected value less the costs of financial distress. As demonstrated in §2, this objective can be represented with a risk-averse utility function.

An increasing difference between the costs of external and internal financing will cause firms to reduce investment if they run short of internal funds (Froot et al. 1993, 1994). This may mean the rejection of a positive-net present value (NPV) project (if it could be financed at the same rate as internal funds) and hence a decrease in the value of the firm. As demonstrated in §3, this phenomenon induces risk aversion over the level of internal funds.

Employees, including directors and officers, may have a substantial fraction of their wealth invested in the firm and do not have a “portfolio of employers” (Treyner and Black 1976, p. 311). Therefore, they may be overinvested relative to the well-diversified shareholder. If these executives are risk averse, they will demand increased compensation in order to bear increases in the uncertainty of their compensation (Stulz 1984) or pursue variance-decreasing actions. For example, May (1995) found a positive relationship between the propensity of CEOs to pursue variance-reducing acquisitions and the fraction of wealth they had invested in the firm. Therefore, increasing uncertainty may interfere with the incentives set up by compensation schemes or make these plans more costly to implement. This imposes a real cost on shareholders and, as demonstrated in §4, induces what could be thought of as risk aversion.

1.2. Decision Analysis

Decision analysis rests on a normative theory of individual decision making. In order to apply this theory within the corporation, decision analysts have viewed the corporation as an entity or single market participant to which preferences and beliefs can be attributed (Luce and Raiffa 1957, p. 13; Howard 1966). Interestingly, while decision analysts have recommended reliance on capital markets to establish corporate time preference (Howard and Matheson 1968), this recommendation has not flowed to the establishment of corporate risk preference. Instead, corporate risk preference is seen as a characteristic to

be measured or assessed from those within the organization (Howard and Matheson 1968). This assessment is “not a direct product of the stockholders,” (p. 669) but instead formalizes judgment “so it can be consistently applied” (Spetzler 1968, p. 668). There is no argument that the corporation should use a particular utility function because, after all, it is a matter of preference. Most importantly, there is no “right” or best utility function (Spetzler 1968). Therefore, corporate utility functions are normative only in the sense that they become a rule or policy; they do not necessarily represent the risk preference the company should use or the utility function that represents shareholders’ preferences. This descriptive view has led to conflicting recommendations regarding corporate risk tolerances, as demonstrated below.

Based on assessments of three companies in the oil and chemical industry, Howard (1988) suggests initially setting corporate risk tolerance (R) equal to one-sixth of equity book value (E_B), where $u(x) = -\exp(-x/R)$. Howard’s associates, McNamee and Celona (1990), augment his suggestion with the companies’ equity market value (E_M) and suggest that risk tolerance may also be set to one-fifth of E_M . The data underlying Howard’s rules of thumb are presented in Table 1.

Note that the market-to-book ratios (M-B) for these companies were all less than one. By comparison, ExxonMobil’s current M-B is over 3.0, and the average M-B for the 30 companies comprising the Dow Jones Industrial Average (DJIA) is currently around 3.6 (or $E_B \approx 0.25E_M$). Therefore, one-sixth E_B and one-fifth E_M can no longer hold simultaneously for the companies in Howard’s study, or more generally.²

In contrast to Howard’s rules of thumb, Walls et al. (1995) suggest, based on a set of direct assessments, that the risk tolerance of an exploration business unit of an oil and gas company is approximately equal to one-fourth of its annual exploration budget.

It is instructive to apply these different rules of thumb to three oil and gas companies: ExxonMobil, ChevronTexaco, and ConocoPhillips. Assume these

² Howard’s assessments were made in the mid-1970s (personal communication). M-B ratios in the oil and gas industry fell below one in 1973 and remained depressed throughout the remainder of the decade.

Table 1 Howard's Risk Tolerance Study

Measure (\$ millions, except ratios)	Company			Average
	A	B	C	
Assessed risk tolerance	150	1,000	2,000	1,050
Equity book value	1,000	6,500	12,000	6,500
Equity market value	940	4,600	9,900	5,147
Market-to-book ratio	0.94	0.71	0.83	0.82
Risk-tolerance/equity book value	0.15	0.15	0.17	0.16
Risk-tolerance/equity market value	0.16	0.22	0.20	0.19

companies' exploration budgets are equal to their exploration expenses. Table 2 details the 2005 exploration expenses, end-of-year (EOY) equity book values, EOY equity market values, and implied risk tolerances for these three companies. For ExxonMobil, currently the largest company in the world with \$340 billion in annual revenue, one-fifth E_M produces a risk tolerance of \$70.4 billion, while one-sixth E_B yields \$18.5 billion, and one-fourth budget yields \$241 million. These differences are significant and could result in different recommendations for action.

Two studies depart from the descriptive view of corporate risk preference. Walls and Dyer (1996) suggest that the selection of the appropriate corporate risk attitude should lead to "superior firm performance" and conversely, selection of an inappropriate corporate utility function should be associated with "lower returns and/or financial distress" (p. 1008). In addition, Walls and Dyer posit that risk tolerance increases with firm size, but at a decreasing rate, and suggest a functional form of $R = aS^b$ or $\log(R) = b \log(S) + \log(a)$, where $0 < b < 1$ and S is a measure of firm size. Smith (2004) places bounds on a firm's risk tolerance by ensuring it is consistent with individual shareholder preferences based on a risk-sharing framework. These bounds do not support the rules of thumb discussed above, and Smith concludes that

Table 2 Comparison of Risk Tolerance Rules of Thumb (\$ Billions)

Company	2005 expl. expense	2005 EOY equity values		Risk tolerance		
		Book	Market	Budget/4	E. book/6	E. market/5
ExxonMobil	0.964	111	352	0.241	18.5	70.4
ChevronTexaco	0.743	60	124	0.186	10.0	24.8
ConocoPhillips	0.661	53	81	0.165	8.8	16.2

large corporations "should be essentially risk neutral towards all but the largest of unsystematic risks" (p. 125).

1.3. Contribution

Jensen and Meckling (1976) argue that corporations are

... legal fictions which serve as a nexus for a set of contracting relationships among individuals... the personalization of the firm implied by asking questions such as "what should be the objective function of the firm" [read: utility function]... is seriously misleading. *The firm is not an individual* [emphasis in original]. (p. 311)

Corporations do not have utility functions, per se. However, as we will demonstrate, what could be thought of as a corporate utility function is induced by both real and implied contracts between shareholders (via the corporation) and financial markets, suppliers, customers, and employees.

The approach we take in this paper is normative. We accept the premises advanced by corporate finance that the objective of public corporations is to maximize shareholder value and that shareholders should value diversifiable uncertainties at their expected value. We then point out that if the motivations to manage risk discussed in §1.1 are true costs to shareholders, then a failure to model these costs, or a decision not to, is a failure to fully model the prospects facing shareholders. As a result, the value measure is modeled incompletely, and thus it may appear that shareholders are risk averse over the incomplete value measure. Finally, we explicitly model the costs of financial distress, costly external finance, and principal-agent issues and determine what degree of risk aversion would account for these effects.

For example, assume shareholders' uncertain wealth, \tilde{w} , is a complex function of a project payoff \tilde{x} . Shareholders' induced utility function (Keeney and Raiffa 1976, p. 56) over x is $u_x(x) = E_{\tilde{w}|x}[u_w(\tilde{w})]$, where $E_{\tilde{w}|x}$ denotes the expectation operator using the conditional distribution over \tilde{w} given that $\tilde{x} = x$. If the relationship between \tilde{w} and \tilde{x} is sufficiently complex, we may choose not to model it explicitly. Instead, we may decide to directly assess a "corporate" utility function over x with the intent of summarizing the complex relationship between \tilde{w} and \tilde{x} .

The view advanced in this paper is that *corporate utility functions can be thought of as modeling shortcuts*. We believe this view closely aligns with decision analysis practice. For example, Walls and Dyer (1996) cite some of the motivations for risk management presented in §1.1 as supporting the existence of corporate utility functions. However, based on the author's experience as a consultant with Strategic Decisions Group, rarely, if ever, do the details discussed in §1.1 appear in decision-making models. This is not to say that leaving out such relationships is a mistake. On the contrary, it is reasonable for decision analysts to limit the level of detail in decision-making *models*, whose goal is to provide clarity of action—not financial valuation.

One never completely models any decision facing shareholders, and therefore one is always dealing with an incomplete value measure. We believe this distinction is at the heart of the disagreement between corporate finance and decision analysis. The theories of corporate finance, which are constructed directly from models of shareholder preferences, assume that the prospects (e.g., wealth impacts) facing shareholders have been *completely* modeled. Decision analysis, on the other hand, with its reliance on direct assessment, does not entail this assumption.

The contribution of this paper is three-fold. First, we suggest and develop a new line of attack on the specification of a corporate utility function. Second, we roughly quantify the degree of risk aversion induced by three oft-cited motivations for risk management. Third, we compare these results to published risk tolerance rules of thumb, which yields partial support of Howard's one-fifth E_M rule of thumb.

We focus on the large corporations with which decision analysts typically work, such as those comprising the S&P 500. While the framework we present is applicable to all firms, our numerical examples may not apply to private or to closely held public corporations.

The remainder of the paper is organized as follows. Section 2 develops an induced utility function based on the costs of financial distress. Section 3 presents a model for costly external financing and the resulting utility function. Section 4 estimates the risk aversion induced by the agency relationship between CEOs and shareholders. Section 5 concludes and recommends areas of future research.

2. Financial Distress

Financial economists generally distinguish the costs of financial distress as being either direct or indirect. Direct costs are payments to parties other than bondholders and shareholders, such as attorney fees and court costs. These costs are incurred only if the firm becomes insolvent. As discussed in §2.3, the direct costs of financial distress, especially the expected direct costs, are believed to be small. Indirect costs are borne by the firm *ex ante*, even if it ultimately avoids financial distress. Examples include lost sales and increased financing costs. The remainder of this section describes several indirect costs, with §2.3 summarizing the literature regarding the magnitude of these costs.

Financial distress, or the threat of it, may cause the corporate contractual system to break down (Novaes and Zingales 1993). For example, employees, including officers and directors, may suffer real costs if the firm goes bankrupt (Rose-Ackerman 1991). These could include the cost of searching for a new job or a decrease in the value they can capture in the labor market because their reputation has been affected. If employees have other employment options, shareholders may have to compensate them *ex ante* for bearing these risks.

If the probability of bankruptcy for a particular firm is increased and customers are unable to fully diversify this risk, they may decrease the price they are willing to pay for the firm's product or not purchase it at all. To increase sales, firms may offer very attractive warranties or pricing, as Chrysler did in the late 1970s (Shapiro and Titman 1998) and General Motors does today. This imposes a real cost on shareholders, even if the firm ultimately avoids bankruptcy.

Suppliers may be reluctant to develop specialty inputs for firms that are on the verge of bankruptcy, thereby increasing the cost of production. Perhaps more importantly, suppliers may withdraw the privilege of purchasing inputs on trade credit once a firm encounters trouble (Shapiro and Titman 1998).

Distributors will be reluctant to promote or enter new markets for the product(s) of a troubled firm. Even if a single product caused financial trouble, uncertainty about a firm's future may drastically affect the sales of the firm's profitable and stable products. In addition, specialized products may require

distributors to invest in training of their salesforce—an investment they will be unlikely to undertake for troubled firms.

Debtholders consider the chance the firm will be unable to repay its debts. This probability is a function of the *total* distribution of the firm's income—not just the systematic uncertainties. Furthermore, because of limited liability, troubled firms have an incentive to undertake risky projects at the bondholders' expense (Romano 1993, p. 123) or avoid projects if most of the gains accrue to bondholders (Mayers and Smith 1987). Bondholders realize these adverse incentives exist and draft detailed covenants to protect themselves. The covenants may specify debt-coverage ratios, whether the firm can issue additional debt, or if it can pay dividends. These restrictions represent a loss in option value to shareholders because they reduce the firm's future alternatives. As a firm's financial position worsens, these covenants are likely to become tighter.

2.1. Model Specification

The model developed in this section is inspired by Greenwald and Stiglitz (1988), who demonstrate that the objective of maximizing expected firm value less the expected costs of financial distress is similar to maximizing the expected utility of firm value under the assumption of risk aversion.

Consider a two-period model of a corporation whose market value, debt plus equity, is determined by the nonstochastic value of its assets in place, totaling x , and the uncertain value of its new investments \tilde{a} , which is revealed in Period 1. The firm enters Period 1 with assets totaling x , which are the result of Period 0 investments. For simplicity, assume \tilde{a} is completely diversifiable and that the risk-free discount rate is zero.

Let B represent the value of the firm's debt. The firm will be considered to be in distress in Period 1 if $x + a \leq B$ or, equivalently, $a \leq B - x$. Therefore, the probability of financial distress is $F(B - x)$, where F is the cumulative distribution function for \tilde{a} . If the firm becomes distressed, it suffers distress costs $k > 0$. Therefore, the value of the firm in Period 1 is $v = x + a - k$ if the firm is in distress and $v = x + a$ otherwise. The Period 0 expected firm value given a particular x is $\bar{v} = x + \bar{a} - kF(B - x)$.

2.2. Induced Risk Aversion

Now suppose that in Period 0, instead of the firm's holding a fixed x , the firm is considering the addition of a new project, or gamble, with uncertain value \tilde{x} , to its portfolio. This gamble will be resolved immediately and determines the x the firm takes into Period 1. Assume \tilde{x} is fully diversifiable and that \tilde{a} and \tilde{x} are probabilistically independent. Assuming shareholders are risk neutral for diversifiable changes in firm value, we have $u(v) = v$ and therefore $U(x) = E_{\tilde{v}|x}[u(\tilde{v})] = x + \bar{a} - kF(B - x)$. U is the induced utility function for Period 0 amounts x and includes an adjustment for x 's impact on the expected cost of financial distress. While the derivation of U was based on the direct costs of financial distress incurred in Period 1, the penalty function could also incorporate the indirect costs of financial distress, as long as these costs are proportional to the probability of distress.

The first derivative of U with respect to x is $U'(x) = 1 + kf(B - x)$, where f is the density function for \tilde{a} (see appendix). The second derivative is $U''(x) = -kf'(B - x)$. The induced risk tolerance function is then

$$R(x) = -\frac{U'(x)}{U''(x)} = \frac{1 + kf(B - x)}{kf'(B - x)}. \quad (1)$$

$U'(x) > 0$, and therefore the sign of the induced risk tolerance depends upon the sign of $-U''(x)$, which depends on the slope of $f(a)$ at $B - x$.

2.3. Illustrative Example

In this section, we apply the model derived above to estimate the degree of risk aversion induced by financial distress costs. This requires the estimation of many inputs that have been measured only indirectly, if at all. Therefore, we will consider a range of input values. Thankfully, as we will demonstrate, our inability to specify some inputs does not weaken our conclusion that financial distress costs cannot explain the levels of risk aversion reported in the decision analysis literature.

Company Financial Structure. Assume $\tilde{a} \sim N(1, 0.3)$. In addition, assume that the firm holds debt such that $B = 0.35$. We will investigate the ratio of the induced risk tolerance to firm value, which renders the monetary units irrelevant. Before the addition of project x to

the portfolio, the probability of default is 1.5%.³ This is equivalent to the probability that a BBB-rated company will default within four years (Brady and Bos 2002), a reasonable benchmark given that most public companies with which decision analysts typically work are at least BBB, the lowest investment grade rating. Furthermore, the indirect costs of financial distress, such as lost sales, should tend to manifest themselves within this general time frame (see discussion below).

We will consider a range for σ_A between 0.2 and 0.4. If $\sigma_A = 0.4$, the probability of default is 5.2%, which is nominally equivalent to the probability a BB-rated company would default within three years. At $\sigma_A = 0.2$ the probability of default is 0.06%, which is approximately the probability that a company with the highest credit rating of AAA would default within four years.

Cost of Financial Distress. In an early study, Warner (1977) estimated the direct costs of bankruptcy (payments to parties other than bondholders or shareholders) by studying 11 railroads that went bankrupt between 1930 and 1955. Warner found that these costs amounted to 1% of market value seven years prior to bankruptcy and rose to 5.3% just prior to bankruptcy.⁴ Warner also noted that this percentage was lower for larger firms and concluded that the *expected* direct costs of bankruptcy are likely to be “very small.”

Altman (1984) measured the direct and indirect costs of financial distress, which he defined to include lost sales, lost profits, higher costs of credit, inability of the firm to take advantage of investment opportunities because it cannot obtain financing, and failure of the firm to execute its strategic plan because decisions must be approved by a trustee in bankruptcy. He studied 12 retail firms and 7 industrials that went bankrupt between 1980 and 1982, finding that the direct costs of bankruptcy averaged 4.3% of firm value three years prior to bankruptcy and 6.2% in the year of bankruptcy. The indirect costs were almost

twice as high, averaging 8.1% of firm value three years before bankruptcy and 10.5% of firm value in the year of bankruptcy.

Andrade and Kaplan (1998) studied 31 highly leveraged transactions that later became financially distressed and concluded that the direct and indirect costs of financial distress range between 10% and 20% of firm value (market value of debt plus equity); their most conservative estimate is no more than 23%.

More recently, Hennessy and Whited (2006) estimated the direct and indirect costs of financial distress to be between 8.4% and 15.1%, depending on the size of the firm.

Based on these studies, we will investigate a range of values for k from 10% to 30% of \bar{a} , with a base case estimate of 20%.

Results. *Specific.* If $\tilde{a} \sim N(1, 0.3)$, $B = 0.35$, $k = 0.20$, and $x = 0$, then based on Equation (1), $R(0) = 5.6$. In other words, the induced risk tolerance is almost six times the value of the company’s portfolio. We could approximate this result noting that the risk premium of a gamble \tilde{x} is $\pi(\tilde{x}) \equiv \bar{x} - \hat{x} \approx \sigma_x^2 / (2R)$, where \hat{x} is the certain equivalent of \tilde{x} . For example, assume the firm is considering adding $\tilde{x} \sim N(0, 0.1)$ to its portfolio. In this case, $E[U(\tilde{x})] = 0.99602$. The certain equivalent is found by solving $U(\hat{x}) = E[U(\tilde{x})]$ for \hat{x} , which yields $\hat{x} = -0.00093$. Therefore, $\pi(\tilde{x}) = 0.00093$ and $R \approx 5.4$.

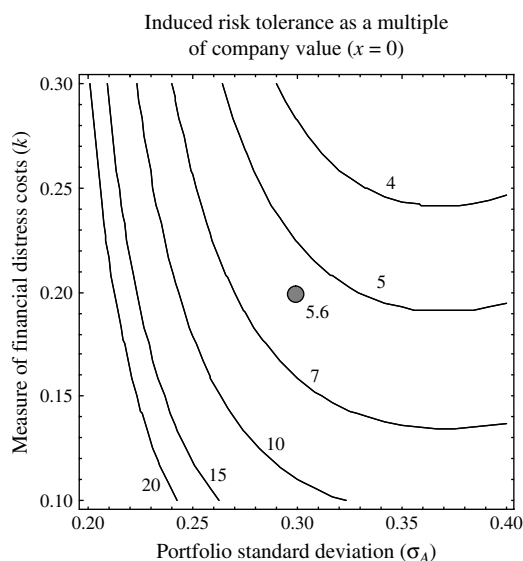
General. Dividing $R(x)$ by \bar{a} yields the induced risk tolerance relative to firm size. Figure 1 displays constant risk-tolerance-to-size-ratio contours as a function of k and σ_A for $x = 0$. Evaluating R at $x = 0$ is reasonable, because most incremental projects are small relative to the existing portfolio. Even projects that are deemed large are unlikely to involve gains or losses of magnitude greater than 10% of firm value. Figure 1 also includes $k = 0.2$ and $\sigma_A = 0.30$.

As shown in Figure 1, decreasing the cost of financial distress increases risk tolerance, as one might expect. In most cases, increasing portfolio variance tends to decrease risk tolerance. The lowest risk tolerance in Figure 1 is four times firm value, which occurs in an extreme scenario where financial distress is more expensive than has been measured empirically, and the firm is in poor financial condition. Given that the example firm is 35% debt on an expected-value basis, four times firm value would be 6.2 times equity

³ Assuming a normal distribution for a does allow for negative values, which are not possible. However, the probability of this is only 0.04% for the assumptions given.

⁴ This increase is due to a decrease in the value of the companies, not an increase in direct bankruptcy cost.

Figure 1 Constant Contours of the Risk-Tolerance-to-Size Ratio



market value, which is more than 30 times larger than Howard’s one-fifth equity market value rule of thumb.

Based on the wide range of values considered in Figure 1, these results suggest that neither financial distress nor the threat of distress can explain the levels of risk aversion reported in the decision analysis literature.

3. Costly External Finance

Information asymmetries between those inside and outside the corporation may result in securities being issued below their *fair value*—the value if all information could be costlessly shared (Myers and Majluf 1984). In this situation, management may decline projects that would have a positive net present value if financed with internal funds. The desire to avoid this “financing trap” induces a preference for internally generated cash. Froot et al. (1994, p. 92) write

The key to creating corporate value is making good investments. The key to making good investments is generating enough cash internally to fund those investments; when companies don’t generate enough cash, they tend to cut investment more drastically than competitors do.

Likewise, Merck’s Chief, Judy Lewent, argues that Merck hedges currencies because of “the potential effect of cash flow volatility on our ability to execute

our strategic plan—particularly, to make investments in R&D that furnish the basis for future growth” (Lewent and Kearney 1990, p. 25). If internal and external funds were perfect substitutes, cash flow volatility would not be a concern. In reality, corporations rely heavily on internally generated funds to cover their cash needs. For example, Brealey and Myers (1991) found that internally generated cash covered 65.7% of cash needs for U.S. nonfinancial corporations between 1969 and 1988.

3.1. Model Specification

In this section, we study the costly external finance model of Froot et al. (1993) and the utility function it induces.

Consider a firm with liquid assets totaling x , facing a perfectly divisible or scalable single-period investment opportunity with a known return. The internal asset level is a result of the firm’s prior investments. An investment of i will gross $g(i, b)$ for certain, where b is a parameter describing the attractiveness of the firm’s investment opportunities. If i exceeds x , the shortfall can be financed with external funds $e(i, x) = i - x$, which can be raised in unlimited amounts, but at additional cost $c(e, k)$ above and beyond the cost of internal funds, where k is a measure of the cost differential between internal and external finance.

In Period 1, the firm chooses its level of investment and thereby selects the level of external financing. The return is earned immediately and all investors are repaid. The shareholders, via the firm, face the following decision:

$$\max_i [g(i, b) - i - c(e(i, x), k)]. \quad (2)$$

To guarantee a solution and simplify our analysis, we place the following restrictions on the return function, cost function, and the firm’s investment opportunities.

ASSUMPTION 1 (A1). $g_1 > 0$: The marginal value of investing is positive for all i .⁵

ASSUMPTION 2 (A2). $g_{11} < 0$: Investment exhibits decreasing returns to scale.

⁵ The subscript denotes the partial derivative of g or c with respect to its first argument.

ASSUMPTION 3 (A3). $c_1 > 0$: The cost of external finance is an increasing function of the amount obtained.

ASSUMPTION 4 (A4). $c_{11} > 0$: The marginal cost of external finance increases with the amount obtained.⁶

ASSUMPTION 5 (A5). $i^* > x$: The firm will always exhaust its supply of internal funds.

This last assumption is made for convenience, but is reasonable because in practice, most companies believe they have more profitable investment opportunities than available capital (e.g., see Walls et al. 1995).

The first-order necessary condition (FONC) corresponding to Equation (2) is $g_1^* = 1 + c_1^*$. Therefore, in the presence of costly external financing, the firm will lower its level of investment from the first-best solution $g_1^* = 1$. The value of the firm for a given x is $x + g^* - i^* - c^*$.

Implicit differentiation of the FONC yields the derivative of i^* with respect to x (see the appendix):

$$di^*/dx = c_{11}^*/(c_{11}^* - g_{11}^*) > 0, \quad (3)$$

where (A2) and (A4) imply the sign of di^*/dx . Increasing (decreasing) the level of internal liquid assets increases (decreases) the optimal level of investment. In other words, the firm's investment decision is sensitive to financing. If internal and external finance were perfect substitutes, di^*/dx would equal zero. The empirical evidence strongly supports $di^*/dx > 0$ (e.g., see Fazzari et al. 1988 and the vast amount of literature that followed).

3.2. Induced Risk Aversion

In the preceding section, we assumed the firm had a known level of internal assets x . Now suppose that at time zero the firm must choose among alternative distributions over x , which could translate to different investment or hedging opportunities. Furthermore, assume these distributions are fully diversifiable. After the firm chooses the optimal distribution, all uncertainty is resolved, the firm proceeds to make

its Period 1 nonstochastic investment decision, and investors are repaid as specified earlier. At the end of Period 1, the value of the firm is $v = x + g - i - c$.

Because different levels of x lead to different investment decisions and different firm values, shareholders will have an induced preference over the level of internal liquid assets. Following corporate finance, we assume shareholders are risk neutral for diversifiable changes in firm value. Therefore, $u(v) = v$ and $U(x) = E_{\tilde{v}|x}[u(\tilde{v})] = x + \max_i [g(i, b) - i - c(e(i, x), k)] = x + g^* - i^* - c^*$. In this case, v is certain, given x , because b is nonstochastic. However, this framework could be applied equally well to the case of uncertain investment returns. We leave this as an area for future research.

The first and second derivatives of U with respect to x are $U'(x) = 1 + c_1^*$ and $U''(x) = g_{11}^*(di^*/dx)^2 - c_{11}^*(di^*/dx - 1)^2 = g_{11}^* \cdot di^*/dx$ (see the appendix). The induced risk tolerance function is

$$\begin{aligned} R(x) &= -U'(x)/U''(x) = -\frac{(1 + c_1^*)}{g_{11}^* \cdot di^*/dx} \\ &= (1 + c_1^*) \left(\frac{1}{c_{11}^*} - \frac{1}{g_{11}^*} \right). \end{aligned} \quad (4)$$

Equation (4) demonstrates how risk aversion is induced by the concavity of c and g . Assumptions (A2) through (A4) imply that $R(x) > 0$.

3.3. Illustrative Example

The previous section developed conditions under which firms should not be risk neutral over the supply of internal liquid assets. These conditions provide a foundation for corporate utility. However, determining the degree of risk aversion implied by Equation (4) for real companies is challenging because the functional forms of g and c are known only imprecisely, if at all. A common assumption for the production function is $g = i^b$ ($0 < b < 1$), where b could be related to financial returns. Even if a functional form was specified for c , only one study has directly measured the marginal cost, direct and indirect, of external equity finance (Hennessy and Whited 2006) and none have measured the indirect cost of debt financing. Therefore, in this section, we make as few assumptions as possible and develop a range of implied risk tolerances. In addition to the information supplied by Hennessy and Whited, we will relate the cost of

⁶ Kaplan and Zingales (1997) note that this assumption is reasonable. Froot and Stein (1998) also detail several microeconomic rationales to support this assumption. As shown in Figure 2, recent work by Hennessy and Whited (2006) and Altinkihc and Hansen (2000) lends support to this assumption.

external finance to studies that have measured di/dx , which is an indirect measure of financing constraints, and studies that have analyzed the impact of outside financing on share price.

Cost of External Finance. The cost of external finance has direct and indirect components. The direct component is largely comprised of transaction costs that are incurred as part of a security offering (e.g., underwriting fees). The indirect component, which is much harder to quantify, stems from information asymmetries between those inside and outside the firm. Myers and Majluf (1984) argue that these information asymmetries will result in securities being offered below their fair value. For example, if executives have information that shareholders do not, they may be able to adversely select the timing of security issues (e.g., issuing stock when they believe it is overvalued). If perfect information sharing is costly, shareholders will reduce the price they are willing to pay for the firm's securities and thereby impose a true cost on the firm.

Smith (1978) found that the costs for underwritten equity offerings, which comprise 90% of all equity issues, averaged 6.17% of proceeds. However, these costs were lower for issues between \$100 and \$500 million, averaging 3.95%. Eckbo (1986) found that the underwriter spread for debt issues was 0.61%. More recently, Altinkihc and Hansen (2000)—hereafter AH—estimate underwriter fees for equity and bond offerings. They estimate that marginal cost of underwriting fees for equity issues is 5.1% for the first \$1 million and increases by 0.03% for every million dollars thereafter. For example, the marginal cost on the last million dollars raised as part of a \$100 million offering would be 8.1%. For bond offerings they estimate that the marginal cost is 1.7% for the first \$1 million and increases by 0.002% for every million dollars thereafter. Therefore, the marginal cost of the last million dollars in a \$100 million debt issue would be approximately 2.0%.

The indirect costs are much more difficult to estimate. Mikkelson and Partch (1986) analyzed the impact of public security issues on firms' equity market value, and their results are summarized in Table 3.⁷

⁷ We do not include private security placements. Private placements tend to be used by smaller firms with limited access to security markets (Krishnaswami et al. 1999).

Table 3 Impact of Security Issuance on Share Price (Average Values)

	Stock		Debt	
	Common	Preferred	Straight	Convertible
Offering size (1982 \$ millions)	39	107	153	75
Offering size (2005 \$ millions)*	96	263	377	185
Offering size/equity market value (%)	15.1	25.6	30.0	22.4
Decrease in equity market value (%)	3.56	0.26	0.23	1.97
Decrease as % of offering (%)	23.6	1.0	0.8	8.8

*Based on an average annual inflation rate of 4%.

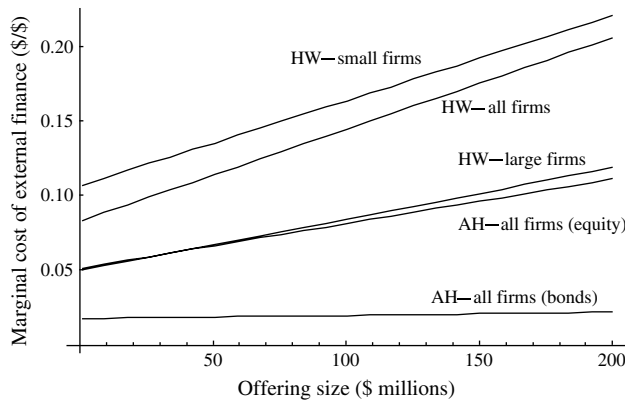
The average debt issue was \$377 million (2005 \$), and 0.8% of the proceeds were lost as a reduction in E_M . Stock offerings were generally quite small, and almost 24% of the issue was lost as a reduction in E_M . These findings are consistent with those of other studies. For example, see Dann and Mikkelson (1984), Asquith and Mullins (1986), Masulis and Korwar (1986), and Eckbo (1986). These studies also analyzed security offerings by public utilities and found a slightly less pronounced effect than for public corporations. For more detail and summary, see Smith (1986).

It is doubtful that the announcement effects detailed in Table 3 directly measure the indirect cost of external finance. For example, the security announcements may merely signal information regarding the firm's prospects (Ross 1977, Miller and Rock 1985), information that would have been revealed eventually. To remedy this situation, Hennessy and Whited (2006)—hereafter HW—attempt to infer both the direct and indirect costs of equity issues (not debt) via a simulation procedure. A summary of their results and those of AH are presented in Figure 2.

HW's estimates are less than what is implied by the announcement-effect studies detailed in Table 3, which they view as being monotonically related to the cost of external finance, but not a direct measure.⁸ For example, HW's marginal cost estimates for a \$100 million issue range from 8.4% (large firms) to 16.3% (small firms). It is interesting that HW's cost estimate for large firms, which includes direct and indi-

⁸ Toni Whited, personal communication with author August 26, 2006.

Figure 2 Estimated Marginal Cost of External Finance



rect costs, is very close to that of AH, which includes direct costs only. This seems to imply that the indirect costs of external finance are small for larger firms (Whited 2006).

Because of the significant uncertainty surrounding the estimates of these costs, we will investigate a range of sensitivities. As the reader will see, this uncertainty does not detract from our conclusion that costly external finance cannot explain the levels of risk aversion reported in the decision analysis literature.

Investment Sensitivity. Fazzari et al. (1988), hereafter FHP, studied U.S. firms between 1970 and 1984 and estimated their investment sensitivity (di/dx). They divided firms into three classes based on an a priori measure of financing constraints, which they took to be the retention ratio (one minus the ratio of dividends to net income). Class 1 firms had a retention ratio of more than 90%; Class 2 firms, between 80% and 90%; and Class 3 firms, less than 80%. Their findings are summarized in Table 4.

Table 4 FHP Investment Sensitivity of U.S. Firms

	Class 1	Class 2	Class 3
Average retention ratio	0.94	0.83	0.58
Average capital stock (1982 \$ millions) (\$)	320	653	2,191
Average capital stock (2005 \$ millions)* (\$)	789	1,609	5,400
Investment sensitivity range (di/dx)	0.46 to 0.67	0.31 to 0.36	0.19 to 0.25

*Based on an average annual inflation rate of 4%.

For comparison, the average retention ratio for the 30 companies that comprise the DJIA was 0.61 in 2004. Similarly, the average retention ratio for the S&P 500 has averaged about 0.70 over the last 10 years. Therefore, Class 3 firms are representative of the large firms with which many decision analysts typically work. For these firms, FHP found that di/dx was between 0.19 and 0.25. FHP do not claim that this investment sensitivity is optimal, but it seems unlikely that such large and established firms facing intense competition in the product and capital markets would have investment and financing policies persistently far from optimal.

Results. Specific. Assume $c = ke^2$ with $k = 0.3$, $g = i^b$ with $b = 0.6$, and $x = 0.05$. Solving Equation (2), we obtain $i^* = 0.22$, $e^* = 0.17$, $c_1^* = 0.10$, $c_{11}^* = 0.6$, $g_{11}^* = -2.0$, and $di^*/dx = 0.23$, which, based on Equation (4), yields $R(0.05) = 2.4$. Or, risk tolerance is 14 times larger than the optimal amount of external finance. If, based on Table 3, we assume e^* is 30% of equity market value, then the induced risk tolerance is more than 4.2 times equity market value.

As was the case in the financial distress model, we could approximate this result using certain equivalent behavior. For example, assume that the firm faces a gamble with a 50-50 chance of $x = 0.0$ or 0.1 . Numerically solving for the certain equivalent, which requires solving Equation (2) multiple times, we obtain $\hat{x} = 0.0495$. Assuming constant risk aversion, we obtain a risk tolerance of approximately 2.4, matching the previous calculation, which was exact.

General. In this section we will rely on empirical studies regarding the costs of external finance and investment sensitivity. From Equation (3), we have $-g_{11}^* di^*/dx = c_{11}^* (1 - di^*/dx)$. Substituting this into Equation (4), we obtain

$$R(x) = \frac{(1 + c_1^*)}{c_{11}^* \cdot (1 - di^*/dx)} \tag{5}$$

Let $c_{11} = c_1/e$ (this relationship holds for $c = ke^2$, for example) and $e = f \cdot E_M$, where $0 < f < 1$. Substituting these into Equation (5) and dividing through by E_M , we obtain the risk tolerance relative to equity market value

$$\frac{R(x)}{E_M} = \frac{f \cdot (1 + c_1^*)}{c_1^* \cdot (1 - di^*/dx)} \tag{6}$$

Figure 3 Sensitivity of Costly-External-Finance Induced Risk Tolerance to Model Parameters (Multiple of Equity Market Value)

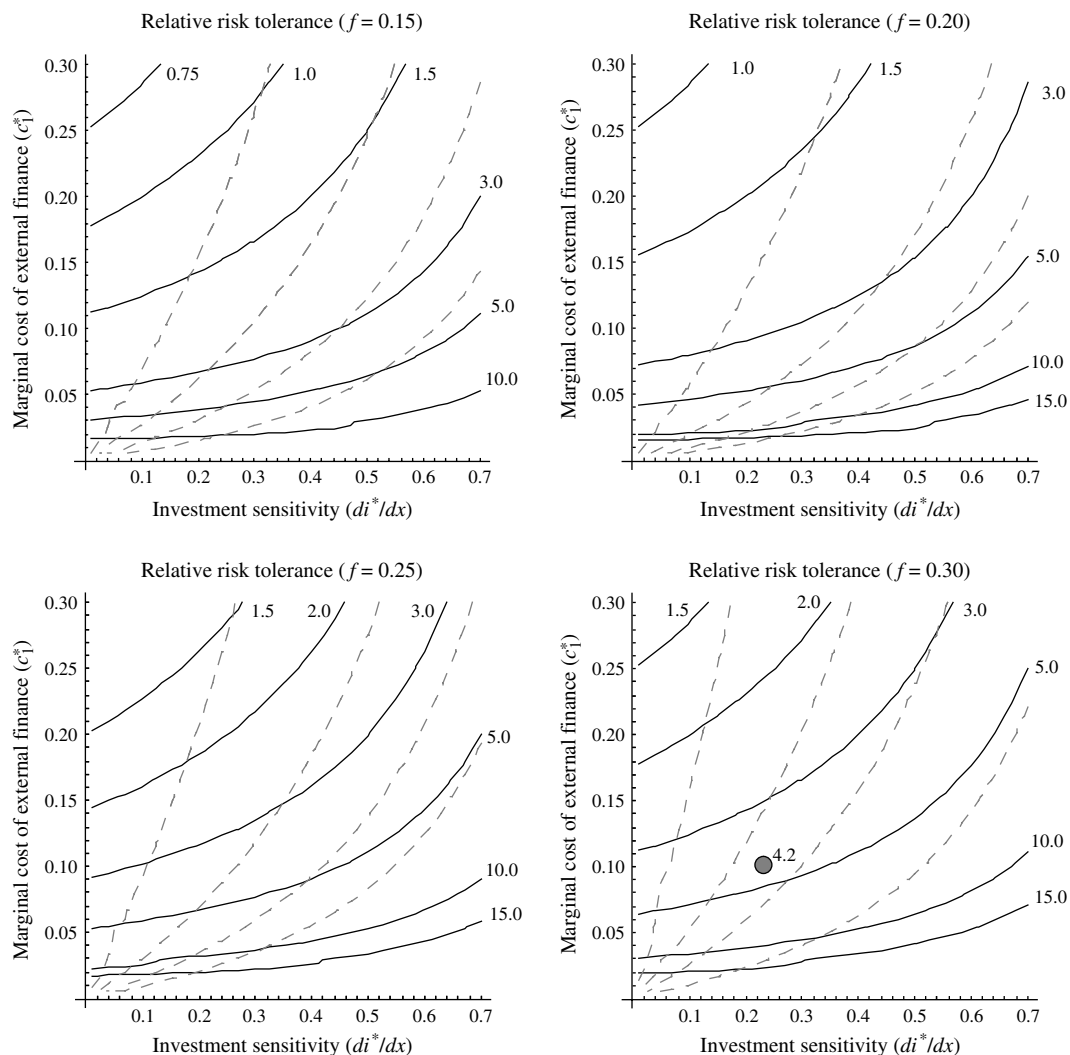


Figure 3 displays $R(x)/E_M$ contours for $f = 0.15, 0.20, 0.25,$ and 0.30 (based on the range in Table 3) as a function of c_1^* and di^*/dx . The gray dashed lines in Figure 3 represent constant quality-of-investment opportunities, as defined by $g_{11}^* = c_{11}^*(1 - (di^*/dx)^{-1})$. To examine the effect of changes in investment sensitivity alone, movement must be along one of these dashed contours. Therefore, as can be seen in Figure 3, decreasing di^*/dx or c_1^* increases risk tolerance. We have added the result from our specific case, discussed above, to the graph with $f = 0.30$.

Consider the issuance of straight debt. According to Table 3, the average issue size was \$0.377 billion, which was 30% of equity market value. According to

AH (see Figure 2), the marginal cost of debt would be approximately 2.6% in this range. Based on this and the investment sensitivities in Table 4, assume $f = 0.30, c_1^* = 2.6\%$, and $di^*/dx = 0.2$. As can be seen in the lower-right graph of Figure 3, these parameters result in a risk tolerance of almost 15 times E_M . Higher values for c_1^* do induce more risk aversion, but never near one-fifth E_M .

According to Figure 2, issuances of stock appear to be much more expensive, with the marginal cost being somewhere between 8% and 16% for a \$100 million offering. However, these offerings were generally smaller, being about 15% of E_M . If we take $f = 0.15, c_1^* = 16\%$, and $di^*/dx = 0.2$, we see in the upper-left

graph of Figure 3 that we still generate risk tolerances of nearly 1.5 times E_M .

How low could the induced risk tolerances be? If we assume $c_1^* = 100\%$, then according to Equation (6) the induced risk tolerance would be $2f/(1 - di^*/dx)$. If we take $di^*/dx = 0.2$, the lowest contour in Figure 3 occurs when $f = 0.15$ and is equal to 0.375, which is still greater than Howard's one-fifth E_M rule of thumb. Therefore, it is difficult to see how the risk of having to raise external finance, while perhaps a motivation to pursue risk management strategies, can explain the levels of risk aversion reported in the decision analysis literature.

4. Agency

This section develops a principal-agent (P-A) model that demonstrates that shareholders should not value diversifiable uncertainties at their expected value. The argument is straightforward: If risk-averse managers must be motivated by holding a share of the corporation, they are exposed to unwanted risk. In order to provide a market-determined certain equivalent wage, increasing uncertainty means managers must receive greater compensation on an expected-value basis. This is costly to shareholders, whom we assume are risk neutral. As we will demonstrate, this induces a risk preference over project outcomes.

4.1. Model Specification

In this section we study two well-known P-A models and the risk aversion they imply. We begin with a general P-A model and then make several simplifying assumptions to obtain a solution.

General Model. This general formulation is well known (Spence and Zeckhauser 1971, Ross 1973, Holmstrom 1979). Consider a single agent, A , and principal, P , each of whom is assumed to maximize the expected utility of his or her wealth. Let $u_A(w)$ be A 's utility function and $u_P(w)$, P 's utility function. The principal values the outcome, ω , and has delegated some decision-making authority to the agent. $\tilde{\omega}$ is a function of an underlying project uncertainty, \tilde{x} , and the agent's action, a .

A is assumed to have a dislike for action, $\delta(a)$, which we will consider to be a decrease in her wealth. A 's action should be interpreted very broadly—not

just the effort exerted or the number of hours worked. For example, action could represent a reduction in shirking, such as a decision not to purchase a company jet. This action increases \tilde{x} , which is net of these effects, but decreases the agent's personal wealth.

In order to motivate the agent to choose the a that P views as best, P chooses a fee schedule or sharing rule, $\phi(a, x, \omega)$. If P can observe a , it is easy to provide the appropriate motivation—compensate the agent only if she chooses the appropriate action. However, in many cases the principal can only observe ω —not a or x . Hence, the sharing rule can only be a function of ω . The agent's action, a , is known given a particular sharing rule.

The uncertain wealth that A obtains from the P-A venture if she takes action a is $\tilde{w}_A(a) = \phi(\omega(a, \tilde{x})) - \delta(a)$. To simplify the notation, we suppress A 's and P 's initial wealth, but assume that their utility functions are defined to include it. Formally, the agent maximizes $E[u_A(\tilde{w}_A(a))]$ over a . Let a^* be the optimal action and \tilde{w}_A^* the agent's uncertain wealth given the optimal action.

Assume that there exists a market-determined certain equivalent wage, \hat{m} , which is the minimum the agent may receive. Therefore, P must choose ϕ such that $E[u_A(\tilde{w}_A^*)] \geq u_A(\hat{m})$. P 's uncertain wealth is $\tilde{w}_P(\phi) = \omega(a^*, \tilde{x}) - \phi(\omega(a^*, \tilde{x}))$. The principal is assumed to maximize his expected utility and solves the following program:

$$\begin{aligned} \max_{\phi} & E[u_P(\tilde{w}_P(\phi))] \\ \text{s.t.} & E[u_A(\tilde{w}_A^*)] \geq u_A(\hat{m}) \\ & a^* \in \arg \max_a E[u_A(\tilde{w}_A(a))] \end{aligned} \quad (7)$$

Simplified Model. In general, solutions to (7) are quite complex or may not exist. However, the model can be solved if we assume the sharing rule is linear, P and A have constant risk tolerance utility functions, and \tilde{x} is normally distributed with mean \bar{x} and variance σ_x^2 (Spremann 1987).

Let $\phi(\omega) = r + s\omega$, where r is the fixed compensation or rent A receives and s is her share of the outcome. This share is not necessarily comprised of equity alone, but could include bonuses, changes in future compensation, stock options, or increased probability

of dismissal for poor performance. Therefore, s is better thought of as the sensitivity of the agent’s compensation to ω , or her “pay-performance sensitivity” (Jensen and Murphy 1990a, b).

Furthermore, assume that the agent’s utility function is $u_A(w_A) = -\exp(-w_A/R_A)$. As throughout this paper, we assume that the principal (which we take as shareholders) is risk neutral for diversifiable uncertainties. Therefore, $u_P(w_P) = w_P$. Although not specifically required, we assume that the agent is risk averse (i.e., $R_A > 0$).

Assume the agent’s action increases the mean of \tilde{x} by γa , but has no effect on the variance. γ is the agent’s marginal productivity of action. The outcome distribution is then $\tilde{\omega} = \tilde{x} + \gamma a$. Further, assume action a decreases A ’s wealth by $\delta(a) = ka^2$, where $k > 0$. k is a measure of the marginal cost of A ’s action.

Given this formulation, the optimal sensitivity can be shown to be (see the appendix)

$$s^* = \frac{\gamma^2 R_A}{\gamma^2 R_A + 2k\sigma_x^2}. \quad (8)$$

Because $2k\sigma_x^2 > 0$, the optimal sensitivity is positive and less than one. In addition, $ds^*/d\sigma_x^2 < 0$ and $ds^*/dR_A > 0$ (see the appendix); as the agent faces greater uncertainty or becomes more risk averse, she should hold a smaller share of the corporation.

P ’s certain equivalent wealth is (see the appendix)

$$\begin{aligned} \hat{w}_P^*(\tilde{x}) &\equiv \hat{w}_P(\tilde{x}, a^*, r^*, s^*) = \bar{x} - \hat{m} + \frac{\gamma^2}{4k} \frac{\gamma^2 R_A}{\gamma^2 R_A + 2k\sigma_x^2} \\ &= \bar{x} - \hat{m} + \frac{\gamma^2}{4k} s^*. \end{aligned} \quad (9)$$

To determine the effect of σ_x^2 on \hat{w}_P , differentiate (9) with respect to σ_x^2 to obtain

$$\frac{d\hat{w}_P^*}{d\sigma_x^2} = \frac{\gamma^2}{4k} \frac{ds^*}{d\sigma_x^2} < 0. \quad (10)$$

Therefore, increasing variance reduces P ’s certain equivalent wealth—even though he is risk neutral. This induces what looks to be risk aversion over x and demonstrates that diversification can increase shareholder value (Spremann 1987).

4.2. Induced Risk Aversion

In §§2.2 and 3.2, we derived an induced utility function, $U(x)$, calculated its risk tolerance function, and demonstrated that a similar result could be obtained by examining certain equivalent behavior. In the P-A case, we may be tempted to consider the utility of a sure x and set $U(x) = \hat{w}_P^*(x) = x - \hat{m} + \gamma^2(4k)^{-1}$. Unfortunately, in this case, $U^{-1}(E[U(\tilde{x})]) = \bar{x} \neq \hat{w}_P^*(\tilde{x})$, which implies that we cannot represent P ’s risk preferences over x with a utility function. This stems from the fact that the agent’s compensation is a function of σ_x^2 and not simply of x . However, when viewed from the outside, or if the P-A venture were not fully incorporated into our decision-making model, P would appear to be risk averse over x . In order to obtain an estimate of this risk aversion, we summarize this behavior with an exponential utility function. Because \tilde{x} is normally distributed, the risk premium is $\pi(\tilde{x}) = \sigma_x^2/(2R)$, where R is the risk-tolerance coefficient induced by the P-A venture.

Define the risk premium as $\pi(\tilde{x}) = \hat{w}_P^*(\bar{x}) - \hat{w}_P^*(\tilde{x})$. Because $\hat{w}_P^*(\bar{x}) = \bar{x} - \hat{m} + \gamma^2(4k)^{-1}$,

$$\pi(\tilde{x}) = \frac{\gamma^2 \sigma_x^2}{2\gamma^2 R_A + 4k\sigma_x^2}. \quad (11)$$

The P-A literature defines Equation (11) as the agency cost of the P-A venture (Spremann 1987).

Setting $\sigma_x^2/(2R)$ equal to Equation (11), solving for R , and simplifying (see the appendix), we have

$$R = R_A/s^*. \quad (12)$$

Because the optimal sensitivity is positive, P will appear risk averse (preferring) if the agent is risk averse (preferring) within the context of the P-A venture.

Equation (12) is surprisingly simple; the P-A induced risk tolerance is the agent’s risk tolerance divided by the optimal sensitivity. We could have obtained this result by employing the risk-sharing framework developed by Smith (2004), with the interpretation of pay-performance sensitivities as “effective” shares and assigning a weight of one to the agent’s utility function (i.e., the firm’s utility function is just the agent’s). However, we must stress that Equation (12) is not the result of a model where we assume the firm’s utility function should be a scaled version of the agent’s. Rather, the risk premium defined in Equation (11) is a true cost borne

by the principal because (1) he wants the agent to hold a share of the company and (2) the agent is risk averse. This cost and the induced risk aversion are optimal from the principal's perspective. In addition, too much emphasis should not be placed on our framing of the agent as a single individual. One could construct a similar framework where the agent is taken to be employees at large or a team of multiple agents.

4.3. Illustrative Example

Assume that the agent under consideration is the chief executive officer (CEO) of a U.S. corporation. In this section we will use the model developed in §§4.1–4.2 to estimate the implied risk tolerance. While the principal-agent model allows for the calculation of an optimal share, its application is hindered by the need to estimate many difficult parameters such as CEO value creation and disutility for action, and understand how they change with firm size. Bickel (1999, Ch. 9) proceeded along these lines and found optimal sensitivities that were close to what have been measured empirically. Therefore, in the interest of space and ease of exposition, this section relies on descriptive studies of pay-performance sensitivities, denoted by \hat{s} . Our results could be interpreted in two ways. First, we could assume that empirical pay-performance sensitivities are optimal, and therefore the induced risk aversion is also optimal from the shareholders' perspective. Second, if the empirical pay-performance sensitivities are not optimal, the induced risk aversion is what we might expect to assess or observe in practice, even if it is not optimal from the shareholders' perspective.

Pay-Performance Sensitivities. Jensen and Murphy (1990a) studied the compensation of 1,688 CEOs between 1974 and 1986. They defined the pay-performance sensitivity as the dollar change in CEO wealth per \$1,000 change in shareholder wealth. CEO wealth included salary, bonus, deferred compensation, stock value, option value, fringe benefits, and the expected value of dismissal. The median sensitivity was \$3.25 per \$1,000 change in shareholder value, or $\hat{s} = 0.325\%$.

Hall and Liebman (1998)—hereafter HL—investigated CEO compensation in 478 of the largest public companies in the United States from 1980–1994. They

found a mean pay-performance sensitivity of \$25 and a median of approximately \$6.

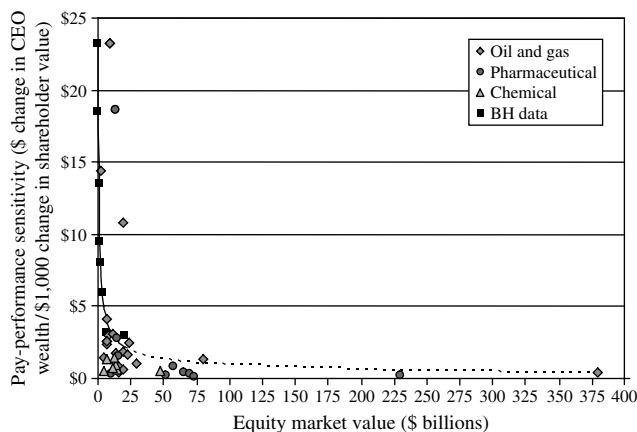
A weakness of the above studies is that they averaged over companies with widely differing sizes and risk characteristics. Recall that the optimal pay-performance sensitivity is a decreasing function of variance. Aggarwal and Samwick (1999) corrected for this by estimating pay-performance sensitivity across a range of variances in firm performance. They studied executive compensation for CEOs and other top executives at 1,500 public companies between 1993 and 1996 and estimated CEO pay-performance sensitivities of \$24.98, \$14.52, and \$4.06 for the 10th, 50th, and 90th percentile variances, respectively. For executives other than the CEO, Aggarwal and Samwick found the corresponding sensitivities to be \$5.47, \$3.29, and \$1.12. These results tend to support the descriptive quality of the principal-agent model because pay-performance sensitivity and variance are inversely related.

Baker and Hall (2004)—hereafter BH—studied how pay-performance sensitivity varies with firm size for corporations with equity market values between \$0.3 and \$20 billion. Unfortunately, they did not provide pay-performance sensitivities for larger companies. However, Schaefer (1998) found pay-performance sensitivities to be approximately proportional to the inverse square root of firm size, using an equation of the form $\hat{s} = k/E_M^{g/2}$, where E_M is equity market value and g was near one. In Figure 4 we plot the BH data and the best fit to this data using the form suggested by Schaefer, with $k = 11.88$ and $g = 1.12$. We extrapolate this best fit from \$20 billion to an E_M value of \$375 billion (the dashed line). The pay-performance sensitivities are in units of dollar change in CEO wealth per \$1,000 change in shareholder wealth.

We add to the data in Figure 4 the pay-performance sensitivities that accrue only from CEO stock holdings, for a sample of companies from the oil and gas, pharmaceutical, and chemical industries, based on data compiled by *Forbes* (2005).⁹ These sensitivities do not include options, bonuses, or the probability of being fired. In general, the sensitivities created

⁹ Companies for which CEO shareholdings were not available were not included. In addition, founding CEOs were excluded because they tend to own a disproportionately large share of their companies and blur the distinction between principal and agent.

Figure 4 Empirical Pay-Performance Sensitivities vs. Equity Market Value

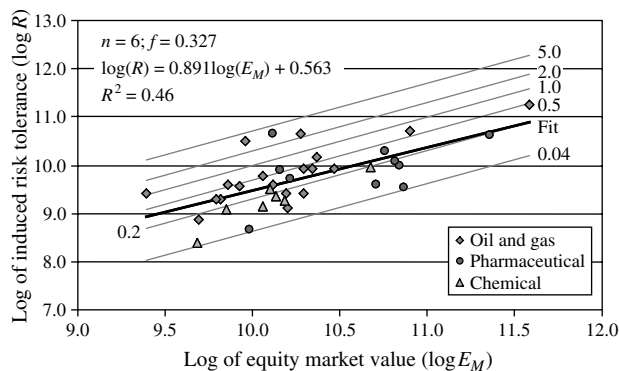


by nonstock/nonoption compensation are small (Hall and Liebman 1998, Aggarwal and Samwick 1999). Options are much more important, providing about twice the pay-performance sensitivity of stock (Hall 1998). Therefore, the stock-only sensitivities underestimate \hat{s} . However, their general trend follows that of the best fit. Therefore, we will rely on the best fit to the BH data in Figure 4 to specify \hat{s} for larger firms.

CEO Relative Risk Aversion. There are two aspects that we must consider in order to determine CEO risk aversion: CEO wealth (as a function of firm size) and CEO relative risk aversion.

In §4.1 we assumed that the agent is risk averse with an exponential utility function and therefore has constant absolute risk aversion. However, the exponential utility function should be considered a local approximation of the agent's, or CEO's, true utility function. Let n be the CEO's relative or proportional risk aversion such that $R_o = n^{-1}w$, where w is the CEO's wealth and R_o is the CEO's risk tolerance. Many economists have estimated n for the general population by investigating the investment decisions of individuals. For example, Friend and Blume (1975) found n to be between two and three. HL noted that "most economists believe [n] to be... in the range of 2 to 4." However, they also listed several references that suggest n is much larger—between 10 and 30. For example, Kandel and Stambaugh (1991) found n to be about 29. Smith (2004) points to a different set of references, with n ranging from about 3 to 25. As a base case, we will assume $n = 6$.

Figure 5 Principal-Agent Induced Risk Tolerance, Base Case

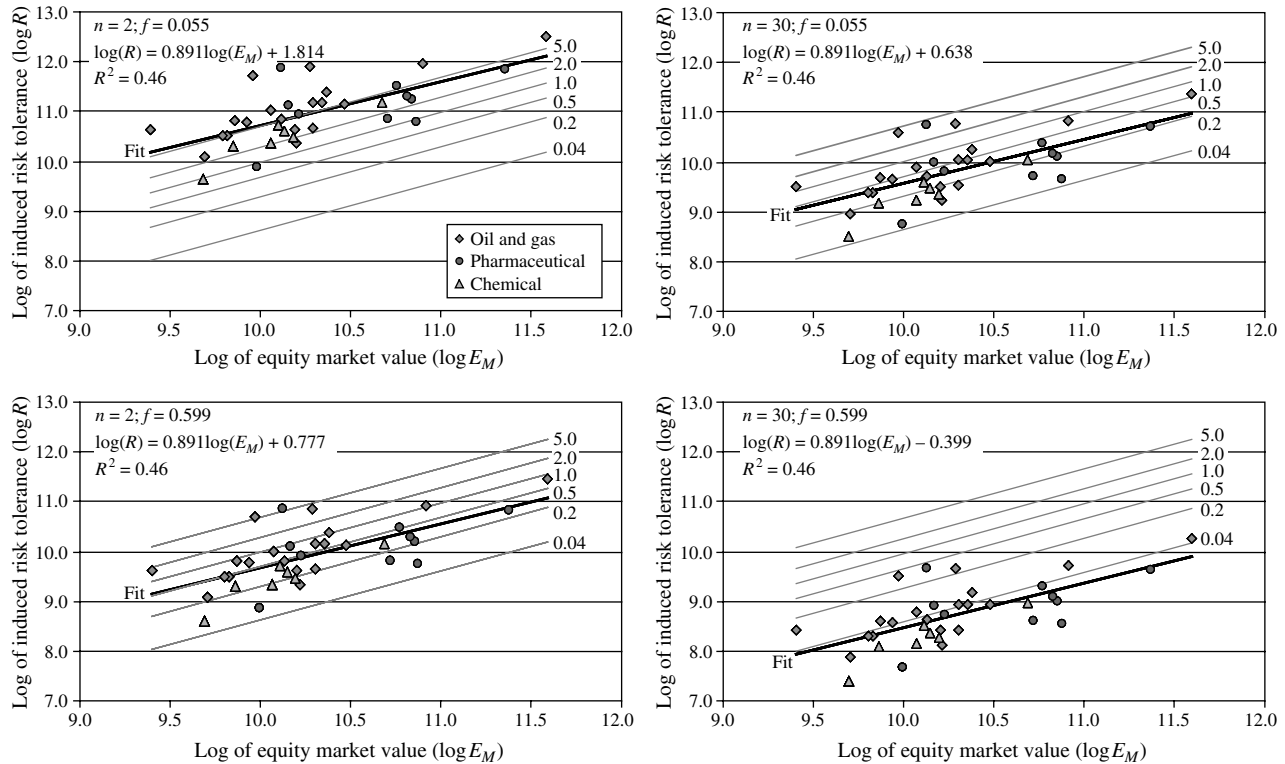


CEO Wealth. As described above, *Forbes* (2005) provides the equity ownership position, e , for over 500 CEOs. We focus on CEOs in the oil and gas, pharmaceutical, and chemical industries. In addition to their firm-specific wealth, CEOs should have accumulated wealth over their career through salary, bonuses, etc. Let f be the fraction of CEOs' current wealth that they hold in the form of company stock. May (1995) estimated that $f = 0.327$ ($\pm 1\sigma = 0.272$) with a median value of 0.240. Let $w_i = e_i/f$ be the wealth of CEO i , where e_i is the equity ownership of CEO i based on the *Forbes* (2005) dataset. The P-A induced risk tolerance for company i is then $R_i = e_i/(f \cdot n \cdot \hat{s}_i)$, where, as discussed above, $\hat{s}_i = 11.88/E_{M,i}^{1.12/2}$ and $E_{M,i}$ is the equity market value for company i .

Results. Figure 5 plots the logarithm of R_i for the reduced *Forbes* dataset, assuming $n = 6$ and $f = 0.327$. The gray upward-sloping lines are risk tolerances as a constant proportion of equity market value (e.g., 0.2 or 20%). A constant risk tolerance of $0.04E_M$ is equivalent to one-fifth E_B if book values are approximately one-fourth E_M . The thick black line is the linear least-squares best fit to the company data. The best fit is of the form $\log(R_i) = z1 \log(E_{M,i}) + z2$, which corresponds to the functional form suggested by Walls and Dyer (1996). If $n = 6$ and $f = 0.327$, then $z1 = 0.891$ and $z2 = 0.563$. Therefore, induced risk aversion increases with firm size as measured by equity market value, but at a decreasing rate.

As can be seen in Figure 5, if $n = 6$ and $f = 0.327$, the induced risk tolerance for some of the companies in this sample is between $0.04E_M$ and $0.20E_M$.

Figure 6 Sensitivity Analysis for Principal-Agent Induced Risk Tolerance



Three companies (two pharmaceutical and one chemical) have risk tolerances close to, but not quite as low as, one-sixth E_B . Therefore, for these parameters, the P-A model can lend some support to Howard’s one-fifth E_M rule of thumb, and perhaps to one-sixth E_B for a few companies. However, the results are highly variable, with some induced risk tolerances on the order of five times E_M .

Figure 6 displays R_i for the cases when $n = 2, 30$ and $f = 0.055, 0.599$ (0.327 ± 0.272). As might be expected, the results are quite sensitive to n and f . For example, if $f = 0.055$ (CEOs are not highly invested in their firms), then induced risk tolerances on the order of $0.2E_M$ are obtained only if CEOs are also highly risk averse ($n = 30$), and no company has risk tolerance as low as $0.04E_M$. Conversely, if $f = 0.599$ (CEOs are highly invested in their firms), then risk tolerances less than $0.2E_M$ are quite common and may even be below $0.04E_M$, unless CEOs are not particularly risk averse ($n = 2$). Lacking definite estimates of n and f , we must conclude that the P-A model lends some support to the one-fifth E_M rule of thumb. However, if

$n \leq 6$ and f is approximately 0.327, this rule of thumb is probably on the lower end of P-A induced risk tolerances; risk tolerances as low as one-sixth E_B are not well supported and prevail only in the most extreme cases.

One may question the normative power of the P-A model and argue that executives should ignore their own preferences and do what is in the best interests of shareholders: being perfect agents. In reality, people are motivated by monetary incentives, as evidenced by the presence of a variable component in virtually all executive compensation packages. Therefore, as long as CEO action is not completely observable, shareholders will find it in their best interests to have CEOs hold a share of the corporation. In efficient labor markets, this will force shareholders to normatively value diversifiable uncertainties below their expected value, as demonstrated here.

5. Discussion and Conclusion

In this paper, we have provided a framework to estimate and bound corporate risk aversion by con-

sidering the degree of risk aversion induced by the costs of financial distress, costly external finance, and the principal-agent relationship between shareholders and CEOs. While these results support the view that corporations should be risk averse over an incomplete value measure, even for diversifiable uncertainties, they suggest that financial distress and costly external finance are unlikely to explain the risk aversion reported in the decision analysis literature. Principal-agent models, on the other hand, do lend some weight to Howard's one-fifth-equity-market-value rule of thumb.

Several extensions to this research are possible and might demonstrate greater risk aversion. For example, the principal-agent model was highly stylized to obtain a solution. Our assumptions of exponential utility and normality might be relaxed, or the agent might be defined as a member of the executive team or any employee of the corporation. As mentioned previously, one could extend the P-A model to include multiple agents. An obvious extension to the costly external finance model is to allow for stochastic investment returns. Perhaps firms with highly uncertain investment opportunities would exhibit greater risk aversion.

We considered the models discussed here in isolation. Ultimately, we would hope to develop a unified theory of corporate risk preference. A first step would be the integration of these models into a single model. Perhaps this will better explain corporate risk aversion.

Another potential source of induced risk aversion is that uncertainty makes it more difficult to optimize operations such as managing supply chains. If this imposes a real cost on firms, then shareholders would prefer to reduce or eliminate uncertainty. This effect could be modeled with an induced utility function if these details are not directly included in the decision-making model. Dynamics might also contribute to risk aversion, as companies hold cash today for future unknown opportunities or risks. For example, debt covenants specify that certain ratios must be maintained, such as the ratio of interest payments to income. This may induce risk aversion over the firm's yearly or quarterly income even if its balance sheet is strong.

Ultimately, we hope this paper serves to deepen our understanding of corporate risk aversion and helps to reconcile the differing approaches taken by the decision analysis and finance communities.

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Appendix

Financial Distress

$U'(x) = 1 - kdp/dx$. Recall, $p = \int_{-\infty}^{B-x} f(a) da$. Applying Leibniz's rule, $dp/dx = -f(B-x)$. Therefore, $U'(x) = 1 + kf(B-x)$ and $U''(x) = -kf'(B-x)$.

Costly External Finance

Sensitivity of Investment to Internal Funds. Let $F(i^*, x) = g_1(i^*) - 1 - c_1(i^* - x) = 0$. By the implicit function theorem, $di^*/dx = -F_x/F_{i^*}$. $F_x \equiv \partial F/\partial x = c_1^*$ and $F_{i^*} \equiv \partial F/\partial i^* = g_1^* - c_1^*$. Therefore, $di^*/dx = c_1^*/(c_1^* - g_1^*)$.

First Derivative of Induced Utility Function. $U(x) = x + g(i^*) - i^* - c(i^* - x)$. $U'(x) = 1 + g_1^* \cdot (di^*/dx) - di^*/dx - c_1^* \cdot (di^*/dx - 1) = 1 - di^*/dx(g_1^* - 1 - c_1^*) + c_1^* = 1 + c_1^*$, where the last equality follows from the FONC.

Second Derivative of Induced Utility Function. Before applying the FONC $U'(x) = 1 - di^*/dx(g_1^* - 1 - c_1^*) + c_1^*$. Therefore, $U''(x) = (d^2i^*/dx^2)(g_1^* - 1 - c_1^*) + (di^*/dx)[g_1^* \cdot (di^*/dx) - c_1^* \cdot (di^*/dx - 1)] + c_1^* \cdot (di^*/dx - 1)$. Applying the FONC, we obtain $U''(x) = g_1^*(di^*/dx)^2 - c_1^*(di^*/dx - 1)^2$. This equation is equivalent to Equation (5) in Froot et al. (1993). Substituting for di^*/dx , we obtain $U''(x) = g_1^*(di^*/dx)$, which is equivalent to Equation (6) in Froot et al. (1993).

Agency

Optimal Sensitivity. The agent's uncertain wealth is

$$\tilde{w}_A(a, r, s) = r + s \cdot \tilde{\omega} - \delta(a) = r + s \cdot (\tilde{x} + \gamma a) - ka^2.$$

Because \tilde{x} is normally distributed, A 's wealth is also normally distributed with mean $\bar{w}_A = r + s \cdot (\bar{x} + \gamma a) - ka^2$ and variance $\sigma_{w_A}^2 = s^2\sigma_x^2$. That A 's utility function is exponential and her wealth is normally distributed allows us to write her certain equivalent wealth as

$$\begin{aligned} \hat{w}_A(\tilde{x}, a, r, s) &= \bar{w}_A - \sigma_{w_A}^2/(2R_A) \\ &= r + s \cdot (\bar{x} + \gamma a) - ka^2 - s^2\sigma_x^2/(2R_A). \end{aligned}$$

Maximization of \hat{w}_A over a yields the optimal level of action $a^* = \gamma s/2k$ and the agent's certain equivalent is $\hat{w}_A(\tilde{x}, a^*, r, s) = r + s\bar{x} + (s^2/4k)(\gamma^2 - 2k\sigma_x^2/R_A)$. If A must receive a certain equivalent market wage of \hat{m} , then her rent must equal $r^* = \hat{m} - s\bar{x} - (s^2/4k)(\gamma^2 - 2k\sigma_x^2/R_A)$. P 's wealth

is $\tilde{w}_p(a, r, s) = (1-s)\tilde{w} - r = (1-s)(\tilde{x} + \gamma a) - r$. Because P is risk neutral, his certain equivalent wealth is $\hat{w}_p(\tilde{x}, a, r, s) = (1-s)(\tilde{x} + \gamma a) - r$. Substituting a^* and r^* into \hat{w}_p yields $\hat{w}_p(\tilde{x}, a^*, r^*, s) = \tilde{x} - \hat{m} + (\gamma^2/4k)(2s - s^2) - s^2\sigma_x^2/(2R_A)$. Maximization of \hat{w}_p with respect to s yields $s^* = \gamma^2 R_A / (\gamma^2 R_A + 2k\sigma_x^2)$.

Derivative of Optimal Sensitivity. The derivative of s^* with respect to the variance is $ds^*/d\sigma_x^2 = -2k\gamma^2/(\gamma^2 R_A + 2k\sigma_x^2)^2 < 0$. To analyze the derivative with respect to R_A , first define the risk aversion coefficient as $\alpha_A = R_A^{-1}$. Then, $ds^*/d\alpha_A = -2k\gamma^2\sigma_x^2/(\gamma^2 + 2k\alpha_A\sigma_x^2)^2 < 0$ or $ds^*/dR_A > 0$.

Principal's Certain Equivalent. Substituting s^* into $\hat{w}_p(\tilde{x}, a^*, r^*, s)$ and simplifying yields

$$\hat{w}_p^*(\tilde{x}) \equiv \hat{w}_p(\tilde{x}, a^*, r^*, s^*) = \tilde{x} - \hat{m} + \frac{\gamma^2}{4k} \frac{\gamma^2 R_A}{\gamma^2 R_A + 2k\sigma_x^2} \quad \text{or}$$

$$\hat{w}_p^*(\tilde{x}) = \tilde{x} - \hat{m} + \frac{\gamma^2}{4k} s^*.$$

Derivation of Induced Risk Tolerance. Setting $\sigma_x^2/(2R)$ equal to Equation (11) and solving for R , we have

$$R = (\gamma^2 R_A + 2k\sigma_x^2)/\gamma^2 = R_A(\gamma^2 R_A + 2k\sigma_x^2)/(\gamma^2 R_A) = R_A/s^*.$$

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