Online Supplement for

Reexamining Discrete Approximations to Continuous Distributions

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This online supplement includes error analysis for the MRO and ZDT shortcut methods, using Pearson's system.

4.2 Extending Keefer and Bodily's Analysis of the Beta Distribution

Table A presents the errors for the MRO and ZDT shortcuts for the ∩-shape beta region, adding to Table 3 of the paper. Although MRO is similar to MCS, it is distinctly more accurate by all measures for both the mean and variance. MRO and ZDT perform similarly although they use very different weights.

Table A. MRO and ZDT errors for the ∩-shape beta distributions.

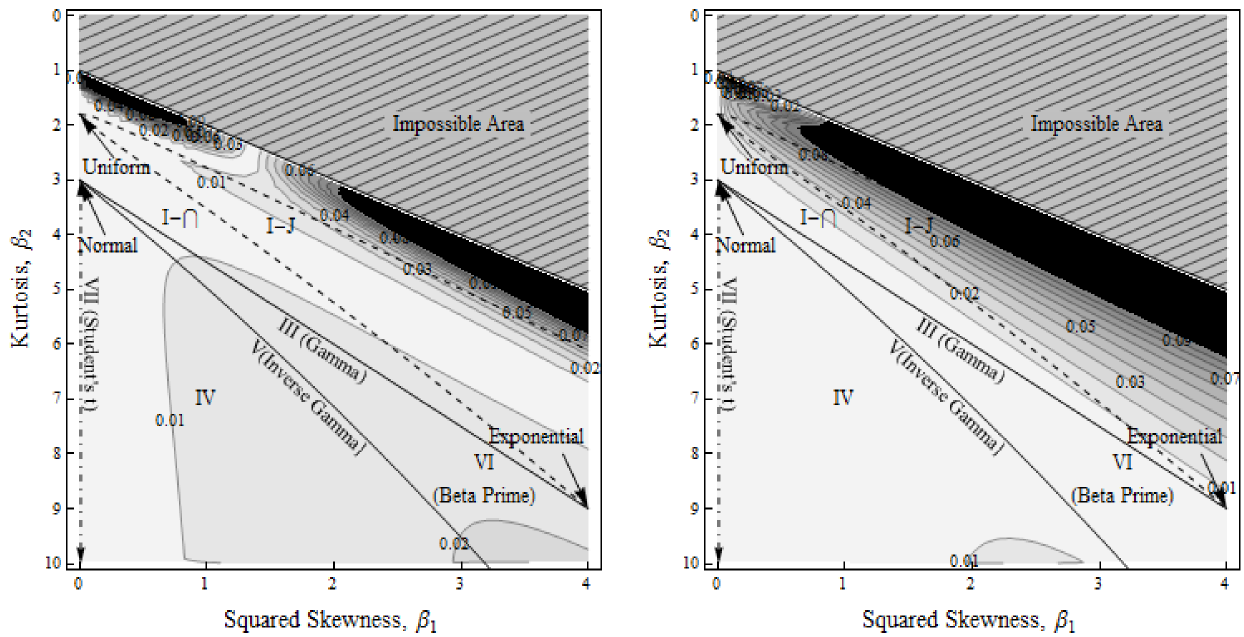
		Mean				Variance			
		AAE	AAPE	ME	MPE	AAE	AAPE	ME	MPE
Beta (∩)	MRO	0.000	0.621	-0.001	-1.647	0.000	9.601	0.002	-20.721
	ZDT	0.001	0.653	0.006	1.919	0.001	8.280	0.016	-21.253

4.3 Expanded Distribution Set Using Pearson's System

Figure A shows plots for the absolute error in the mean for MRO and ZDT over the entire region of Figure 1 in the paper. MRO appears to perform better than MCS, but not as well as ESM. ZDT actually performs quite well outside of the beta region, having absolute errors of less than 0.01 over all of the beta prime and most of the type IV regions in the plot, and generally performing similar to ESM. Note that of the shortcut methods considered in the paper, ESM most resembles ZDT. The ZDT shortcut's higher weighting of the outer points (0.333) may help it account for the tails, with its percentiles that are less extreme than EPT and ZDI.

Figure A. Errors in the mean for the MRO and ZDT discretization shortcuts.

a) MRO **b) ZDT**



Plots of absolute error in the variance for both methods are shown in Figure B. Again, MRO is generally more accurate than MCS, but less accurate than EPT, ZDI, and ESM, and ZDT's performance on the variance is similar to ESM.

Figure B. Errors in the variance for the MRO and ZDT discretization shortcuts.

a) MRO **b) ZDT**

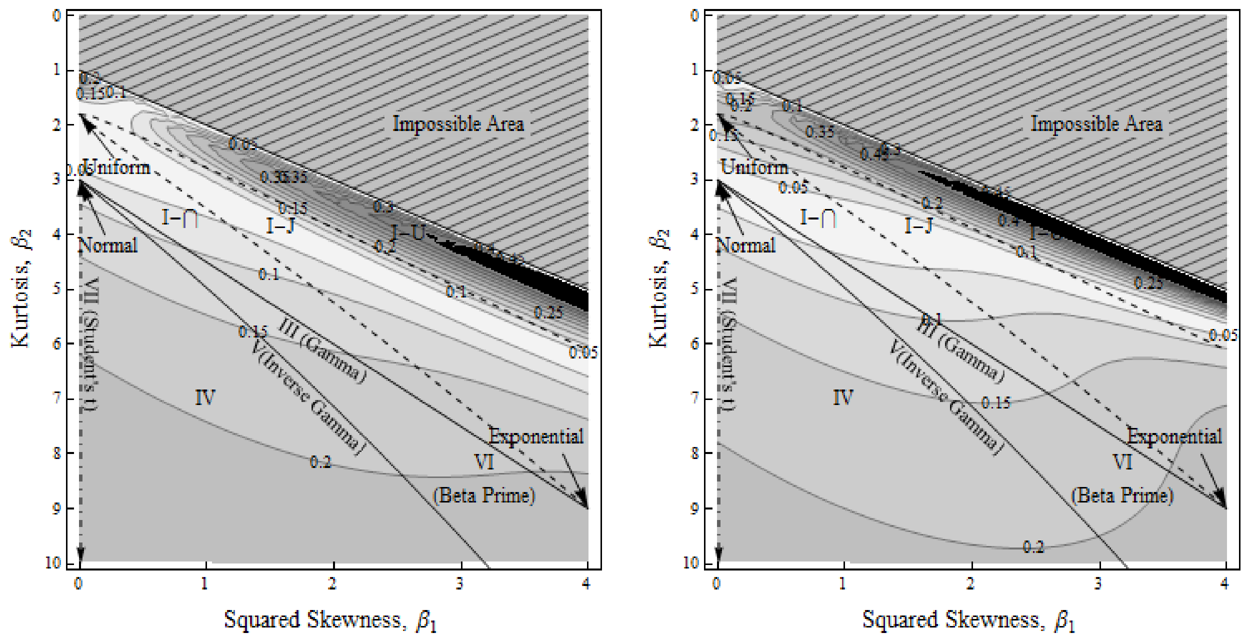


Table B shows the Average Error (AE), Average Squared Error (ASE), and Maximum Error (ME), as defined in Table 2 and Table 5 of the paper, for both MRO and ZDT for the various Pearson regions and transition types. We see that MRO performs better than MCS on all types except the U-shape beta, for which it performs slightly worse. ZDT performs slightly worse than ESM for the gamma and all three beta types, and nearly identically to ESM for the beta prime, inverse gamma, and type IV regions.

Table B. Shortcut method errors for standardized distributions.

		Mean			Variance		
		AE	ASE	ME	AE	ASE	ME
Beta (U)	MRO	0.065	1.47E-02	0.298	0.264	0.104	0.673
	ZDT	0.203	5.76E-02	0.539	0.296	0.133	0.845
Beta (J)	MRO	0.002	1.44E-04	0.048	-0.065	0.012	-0.213
	ZDT	0.048	3.10E-03	0.118	-0.080	0.018	-0.222
Beta (∩)	MRO	-0.008	8.58E-05	-0.017	-0.092	0.012	-0.204
	ZDT	0.009	1.08E-04	0.025	-0.045	0.010	-0.213
Gamma	MRO	-0.013	1.74E-04	-0.017	-0.144	0.023	-0.215
	ZDT	0.003	1.04E-05	0.006	-0.114	0.018	-0.223

Beta Prime	MRO	-0.017	2.99E-04	-0.021	-0.185	0.035	-0.228
	ZDT	-0.002	1.67E-05	-0.009	-0.166	0.030	-0.226
Inverse Gamma	MRO	-0.014	2.22E-04	-0.020	-0.158	0.027	-0.220
	ZDT	-0.005	2.83E-05	-0.009	-0.125	0.019	-0.205
Type IV	MRO	-0.011	1.47E-04	0.020	-0.197	0.040	-0.243
	ZDT	-0.006	4.29E-05	0.010	-0.170	0.030	-0.223